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QUALITATIVE PROPERTIES OF OPTIMUM ACOUSTIC STRUCTURES

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Problems of optimum synthesis of layered structures exposed to acoustic waves are investigated. Qualitative laws governing the interrelationship between the parameters in structures that realize the limiting possibilities for control of the energy characteristics of acoustic waves are established. A numerical example is given.

In recent decades a great deal of attention has been given to the study of wave processes in inhomogeneous structures. A large role is played by layered structures (LS) used to control the energetics of wave processes (WP), whose most important characteristics are the energy coefficients of transmission and reflection, which characterize the fraction of the energy of the wave that passes through the structure (or is reflected from it) as a function of the frequency. In investigation of WP in layered media the problem of construction of a layered structure with the required properties is central. It consists in choosing the structure of a layered medium whose energy characteristics will be closest to the prescribed relations. In solving this problem, we will consider use of the necessary conditions for optimality associated with nonlocal variations of the control parameters [1-3]. To find all the variants of layered structures that realize the limiting possibilities for control of the energetics of a WP, it is necessary to isolate the entire aggregate of solutions that satisfy the necessary conditions for optimality. Just as in the case where methods of synthesis are used that are based on local variation of an admissible solution, the efficiency of singling out all globally optimum variants of layered structures is associated with the extent to which the dependence of the quality functional on the aggregate of the control parameters has a multiextremum character. Results of computational experiments show that the number of variants that satisfy the necessary conditions for optimality in problems of synthesis of LS exposed to wave actions is very significant. Therefore, singling out all the variants of LS that realize the limiting possibilities presents substantial computational difficulties.

With the above in mind, we will consider another path to the investigation of limiting possibilities that is associated with the study of the existence of internal symmetry in the interrelationship between the parameters that compose the optimum structure. The presence of this internal symmetry in problems of synthesis may indicate that the structures that realize the limiting possibilities will group only within a certain narrow compact set Q. A qualitatively new way of compressing the set of admissible variants of structures and developing, on this basis, effective methods of synthesis is associated with investigation of the possibility of singling out a narrow compact set Q that contains the entire aggregate of variants that realize the limiting possibilities.

Let us consider oblique incidence of a plane nonmonochromatic wave on a multilayer system consisting of plane layers, in which shear waves do not propagate. Then the propagation of the acoustic wave in the layered medium can be described by the following boundary-value problem:

$$\dot{f}(z) = \rho(z) g(z), \quad \dot{g}(z) = -\omega^2 \mu [\rho(z)] f(z), \quad 0 \le z \le l,$$

$$g(0) = \frac{ik_{\text{h}}(\omega) \cos \vartheta_0}{\rho_{\text{h}}} (2 - f(0)), \quad g(l) = \frac{ik_{\text{w}}(\omega) \cos \vartheta_{\text{w}}}{\rho_{\text{w}}} f(l). \tag{1}$$

Here $f(z)$ $(0 \le z \le l)$ is the complex amplitude of the acoustic wave, $\rho(z)$ $(0 \le z \le l)$ is the distribution of the density over the thickness of the structure; $k_{\text{h}}(\omega) = \omega/c_{\text{h}}$, $k_{\text{w}}(\omega) = \omega/c_{\text{w}}$; ρ_{h} and c_{h} are, respectively, the density

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and the velocity of wave propagation in the half-space into which the wave passes; ϑ_w is the angle at which the wave emerges from the structure; $\mu(z) = (c^{-2}(z) - c_h^{-2}\sin^2\vartheta_0)/\rho(z)$, $c(z)$ $(0 \le z \le l)$ is the distribution of the acoustic-wave velocity in the structure. The physical parameters of the layered structure are considered to be interrelated by the functional relationship $c = c(\rho)$, making it possible to recover uniquely the acoustic-wave velocity in the material from its density. Here, only the density ρ will be an independent physical parameter. Let the admissible collection Λ consist of just two materials with densities $\bar{\rho}$ and $\bar{\bar{\rho}}$. For each $z \in [0, l]$ the following inclusion is satisfied:

$$\rho(z) \in \Lambda. \tag{2}$$

The energy coefficient of transmission of the acoustic wave is given by the solution of boundary-value problem (1) at $z = l$:

$$T = \frac{c_h \rho_h \cos\vartheta_w}{c_w \rho_w \cos\vartheta_0} |f(l)|^2.$$

We are to design a layered structure that would have high reflection of acoustic waves in some portions of the spectrum and low in others. In a variational formulation the problem consists in minimization of the criterion

$$J = \int_{\omega_{min}}^{\omega_{max}} \tau(\omega) T(\omega)\, d\omega \tag{3}$$

with the additional condition that for some frequency $\omega = \omega^*$ the functional characteristic of the structure must be the maximum attainable:

$$T(\omega^*) = q(\omega^*). \tag{4}$$

Here $\tau(\omega)(-1 \le \tau(\omega) \le 1)$ is the weighting function; $q(\omega^*)$ is the maximum attainable value of the energy coefficient of transmission for the frequency $\omega = \omega^*$.

Let us introduce the set of distributions of the density $Q(f^*)$, $f^* = \omega^*/2\pi$, whose elements satisfy the following system of relations:

$$0 \le \Delta_s \le \frac{1}{2f^* \sqrt{c_s^{-2} - c_h^{-2}\sin^2\vartheta_0}} \quad (s = \overline{1, N}),$$

$$\Delta_s = \Delta_{s-2} \quad (s = \overline{4, N-1}), \quad N_{min} \le N \le N_{max}. \tag{5}$$

Here $[N_{min}, N_{max}]$ is an interval that includes the number of layers of the optimum structure.

We also introduce the set of distributions of the density $Q_0(f^*)$ whose elements satisfy the system of relations

$$0 \le \Delta_1 \le \frac{1}{2f^* \sqrt{c_1^{-2} - c_h^{-2}\sin^2\vartheta_0}}, \quad 0 \le \Delta_N \le \frac{1}{2f^* \sqrt{c_N^{-2} - c_h^{-2}\sin^2\vartheta_0}},$$

$$\Delta_s = \frac{1}{4f^* \sqrt{c_s^{-2} - c_h^{-2}\sin^2\vartheta_0}} \quad (s = \overline{2, N-1}), \quad N_{min} \le N \le N_{max}, \tag{6}$$

$$N_{min} = \left[\frac{8f^* l_{min} \sqrt{(\bar{c}^{-2} - c_h^{-2}\sin^2\vartheta_0)(\bar{\bar{c}}^{-2} - c_h^{-2}\sin^2\vartheta_0)}}{\sqrt{\bar{c}^{-2} - c_h^{-2}\sin^2\vartheta_0} + \sqrt{\bar{\bar{c}}^{-2} - c_h^{-2}\sin^2\vartheta_0}} \right],$$

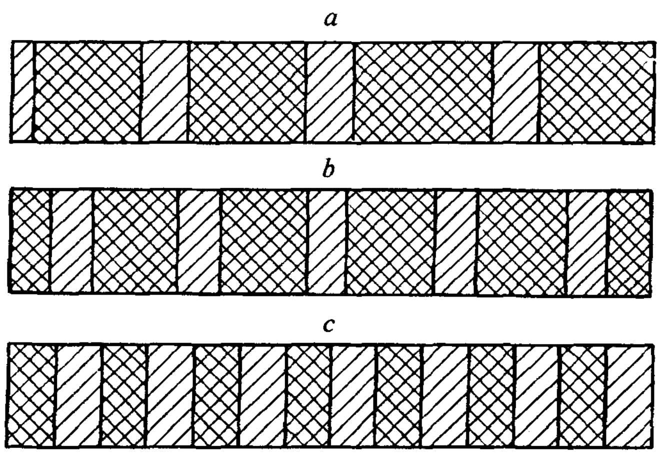

Fig. 1. Resulting structures built following a calculation procedure based on the necessary conditions of optimality (a), complete partitioning on the set $Q_0(f)$ of (6) (b), and complete partitioning on the set $Q(f)$ of (5) (c).

$$N_{\max} = \left[\frac{8f^* l_{\max} \sqrt{(\bar{c}^{-2} - c_h^{-2} \sin^2 \vartheta_0)(\bar{\bar{c}}^{-2} - c_h \sin^2 \vartheta_0)}}{\sqrt{\bar{c}^{-2} - c_h^{-2} \sin^2 \vartheta_0} + \sqrt{\bar{\bar{c}}^{-2} - c_h^{-2} \sin^2 \vartheta_0}} \right] + 4 .$$

Here $\bar{c} = c(\bar{\rho})$, $\bar{\bar{c}} = c(\bar{\bar{\rho}})$.

Analysis of the necessary conditions for optimality in the problem of optimum control (1)-(3) makes it possible to ascertain the following. The aggregate of all the variants of multilayer structures that furnish a global minimum to the quality functional (3) in the problem of optimum synthesis (1)-(4) belongs to the set $Q(f^*)$ if the optimum thickness of the structure l^* is limiting or to the set $Q_0(f^*)$ if $l^* \in (l_{\min}, l_{\max})$. Here $Q_0(f^*) \subset Q(f^*)$.

Thus, with the frequency $\omega = \omega^*$ being known, the initial multiparameter problem of synthesis is reduced to the three-parameter problem of minimization of criterion (3) on the set $Q(f^*)$ of (5) if the optimum thickness of the structure l^* is limiting or to the one-parameter problem of minimization of criterion (3) on the set $Q_0(f^*)$ of (6) if $l^* \in (l_{\min}, l_{\max})$. For this case the results obtained make it possible to solve the problem of synthesis completely, since the aggregate of all globally optimum solutions can be found efficiently.

Let us consider an example. Suppose the admissible collection includes two materials with the physical parameters $\bar{\rho}$ = 1140 kg/m^3, $\bar{c}$ = 2310 m/sec and $\bar{\bar{\rho}}$ = 2700 kg/m^3, $\bar{\bar{c}}$ = 5100 m/sec. The total thickness of the multilayer structure is l = 185 mm. The physical properties of the semi-infinite media that border the structure correspond to the parameters of air $\rho_h = \rho_w$ = 1.29 kg/m^3, $c_h = c_w$ = 331 m/sec. The case of normal incidence of a monochromatic acoustic wave with a frequency of 50 kHz on a multilayer structure is analyzed. It is required that an acoustic system that realizes the limiting possibilities for damping the acoustic effect be designed. The quality of the structure is evaluated by the magnitude of the damping factor $\nu = -20 \log |f(l)|$, which must be maximized ($f(l)$ is the value of the solution of boundary-value problem (1) at $z = l$).

To solve this problem, we consider use of a synthesis procedure that is based on the necessary conditions for optimality and is associated with spicular variation of the admissible solution [3-5] and a technique that is associated with singling out a narrow compact set on the basis of an analytical description of its boundaries. Use of the former makes it possible to construct a multilayer structure with number of layers N^* = 8 (see Fig. 1a). The thicknesses of the layers are (in mm): $\Delta_1^* = 2$; $\Delta_2^* = 33$; $\Delta_3^* = 13$; $\Delta_4^* = 36$; $\Delta_5^* = 13$; $\Delta_6^* = 37$; $\Delta_7^* = 12$; $\Delta_8^* = 39$. (The relationship between the thicknesses of the layers in this figure, just as in subsequent figures, is kept in conformity with the specific values.) The first layer of the optimum structure consists of the first material of the admissible collection. The value of ν^* is equal to 115.45 dB. A single-layer structure consisting of the second material of the admissible collection was taken as the initial approximation. For this structure ν = 83.42 dB. Use of other initial approximations leads to worse solutions. Single-parameter optimization on the set $Q_0(f)$ of (6) leads to a multilayer structure with number of layers N^* = 11 (see Fig. 1b). The thicknesses of the layers of the structure are equal to (in mm): $\Delta_1^* = 13$; $\Delta_2^* = \Delta_4^* = \Delta_6^* = \Delta_8^* = \Delta_{10}^* = 11$; $\Delta_3^* = \Delta_5^* = \Delta_7^* = \Delta_9^* = 26$; $\Delta_{11}^* = 12$. The first layer consists of the second material of the admissible collection. The value of ν^* is equal to 150.0 dB. Three-parameter optimization on the set $Q(l)$ of (5) (the thicknesses of two neighboring inner layers and the thickness of one of the boundary layers are the varied parameters) leads to a multilayer structure with number of layers $N^* = 14$ (see Fig. 1c). The thicknesses of all the layers of the structure are identical: $\Delta_s^* = l/N^*$ $(s = \overline{1, N^*})$. The first layer consists of the second material of the admissible collection. The value of ν^* is equal to 156.07 dB. This multilayer structure is globally optimum. Since in this example the thickness of the structure is fixed, the globally optimum solution belongs to the set $Q(l)$ of (5). Nevertheless, even optimization on the set $Q_0(l)$ of (6), on which only the thickness of one of the boundary layers (the optimum thicknesses of the inner layers can be calculated beforehand) is a varied parameter, allows one to synthesize a layered structure that, not being globally optimum, nevertheless has a reflection that substantially exceeds that provided by a layered structure designed by the calculation procedure based on the Pontryagin necessary conditions for optimality.

Results of numerical experiments show that use of computational procedures of optimization based on the Pontryagin necessary conditions for optimality allow one at least to construct more efficient solutions compared to methods that are associated with local variations of the control parameters, but, nevertheless, by their characteristics these solutions may be considerably inferior to globally optimum ones due to the considerable multiextremality of the wave problems of synthesis.

Thus, the procedure of synthesis associated with singling out narrow compact sets on the basis of an analytical description of their boundaries offers great potentiality for studying the limiting possibilities of layered structures for controlling the parameters of a wave field.

NOTATION

ω, frequency; l, total thickness of the structure; z, coordinate; ϑ_0, angle of incidence; ϑ_w, angle at which the wave emerges from the structure; $\rho(z)$, density function; ρ_h, density of the half-space from which the wave arrives; c_h, speed of wave propagation in the half-space from which the wave arrives; ρ_w, density of the half-space into which the wave goes after emergence from the structure; c_w, speed of wave propagation in the half-space into which the wave goes after emergence from the structure; i, imaginary unit; Λ, set of the densities of the materials of the admissible collection; T, energy coefficient of transmission; J, quality criterion; $\tau(\omega)$, weighting function; ω_{min}, ω_{max}, lower and upper boundaries of the filtered range of frequencies; N, number of layers; s, number of a layer; Δs, thickness of the s-th layer.

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